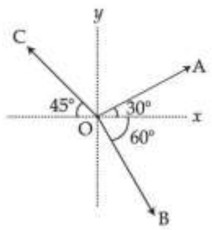
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
26th August Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 The magnitude of vectors $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ in the given figure are equal. The direction of $\vec{OA} + \vec{OB} - \vec{OC}$ with x-axis will be :



Options

1. $\tan^{-1} \dfrac{(1 + \sqrt{3} - \sqrt{2})}{(1 - \sqrt{3} - \sqrt{2})}$

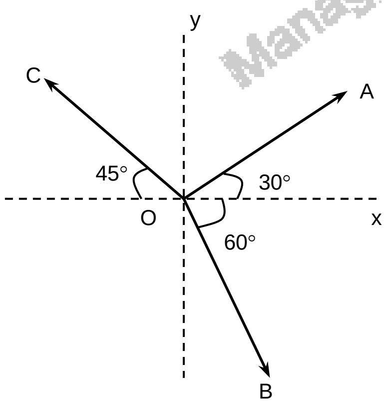
2. $\tan^{-1} \dfrac{(\sqrt{3} - 1 + \sqrt{2})}{(1 + \sqrt{3} - \sqrt{2})}$

3. $\tan^{-1} \dfrac{(\sqrt{3} - 1 + \sqrt{2})}{(1 - \sqrt{3} + \sqrt{2})}$

4. $\tan^{-1} \dfrac{(1 - \sqrt{3} - \sqrt{2})}{(1 + \sqrt{3} + \sqrt{2})}$

Ans: $\tan^{-1}\left[\dfrac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$

Sol:



Let magnitude be equal to λ

$$\overrightarrow{OA} = \lambda\left[\cos 30^\circ \hat{i} + \sin 30 \hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB} = \lambda\left[\cos 60^\circ \hat{i} - \sin 60 \hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC} = \lambda\left[\cos 45^\circ(-\hat{i}) + \sin 45 \hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} = \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{\sqrt{2}}}\right] = \tan^{-1}\left[\frac{\sqrt{2} - \sqrt{6} - 2}{\sqrt{6} + \sqrt{2} + 2}\right] = \tan^{-1}\left[\frac{1 - \sqrt{3} - \sqrt{2}}{\sqrt{3} + 1 + \sqrt{2}}\right]$$

Q.2 The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permitivity of material is 50)

Options
1. 0.9 mA
2. 9.0 μA
3. 0.9 μA
4. 9.0 mA

Ans: 0.9 mA

Sol: $\rho = 200\ \Omega$ m
$C = 2 \times 10^{-12}$ F
V = 40 V
K = 56

$$i = \frac{q}{\rho k \varepsilon_0} = \frac{q_0}{\rho k \varepsilon_0} e^{\frac{t}{\rho k \varepsilon_0}}$$

$$i_{max} = \frac{2 \times 10^{-12} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}} = \frac{80}{10^4 \times 8.85} = 903\mu A = 0.9 mA$$

Q.3 An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90 W. How long it would take to increase the internal energy by 2.5×10^3 J ?

Options
1. 4.1×10^1 s
2. 2.5×10^2 s
3. 2.4×10^3 s
4. 2.5×10^1 s

Ans: 2.5×10^2 s

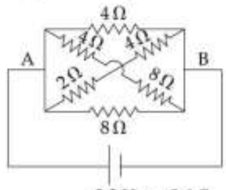
Sol: $\Delta Q = \Delta U + \Delta W$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

$$\frac{6000}{60}\frac{J}{sec}=\frac{2.5\times10^{3}}{\Delta t}+90$$

Δt = 250 sec

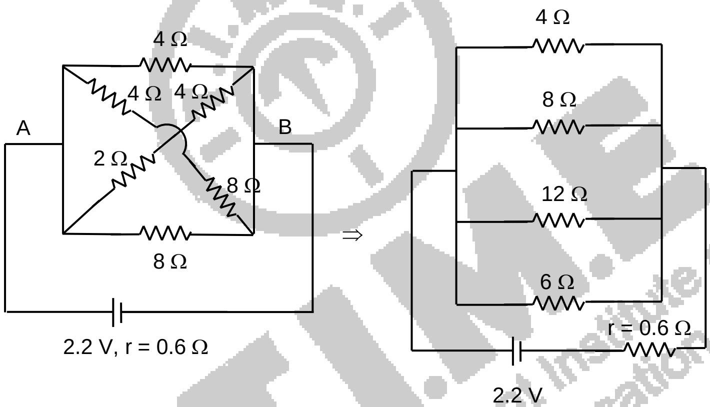
Q.4 In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6 Ω. Calculate the power dissipated in the whole circuit :



Options
1. 2.2 W
2. 1.32 W
3. 4.4 W
4. 0.65 W

Ans: 2.2 W

Sol:



$$\frac{1}{R_{eq}}=\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{6}=\frac{6+3+2+4}{24}=\frac{15}{24}$$

$$R_{eq}=\frac{24}{15}=1.6\Rightarrow R_T=1.6+0.6=2.2\ \Omega$$

$$P=\frac{V^2}{R_T}=\frac{(2.2)^2}{2.2}=2.2\text{ W}$$

Q.5 A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance $X_L = 250\pi\ \Omega$ and an unknown capacitor. The value of capacitance to maximize the average power should be :
(take $\pi^2 = 10$)

Options
1. 25 μF
2. 400 μF
3. 4 μF
4. 40 μF

Ans: 4μF

Sol: $X_L = X_C$

$$250\pi = \frac{1}{2\pi(50)C}$$

$C = 4 \times 10^{-6}$ F

Q.6 In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20^{th} division of the circular scale coincides with reference line. Calculate the true reading.

Options
1. 5.20 mm
2. 5.15 mm
3. 5.00 mm
4. 5.25 mm

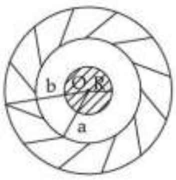
Ans: 5.15 mm

Sol: Least count (L.C) = $\frac{0.5}{50}$

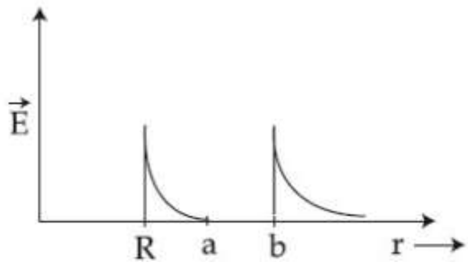
True reading = $5 + \frac{0.5}{50} \times 20 - \frac{0.5}{50} \times 5$

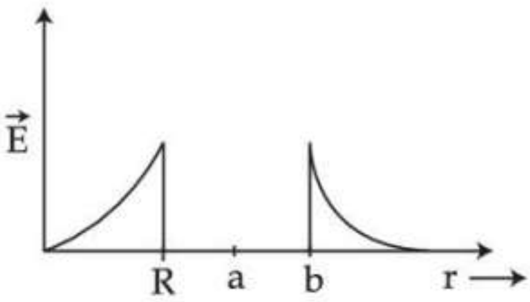
$= 5 + \frac{0.5}{50}(15) = 5.15\text{ mm}$

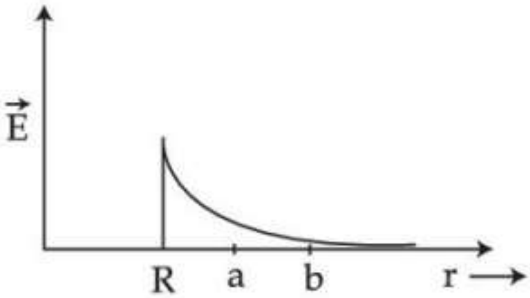
Q.7 A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field $\vec{E}$ as a function of distance r from centre O is given by :

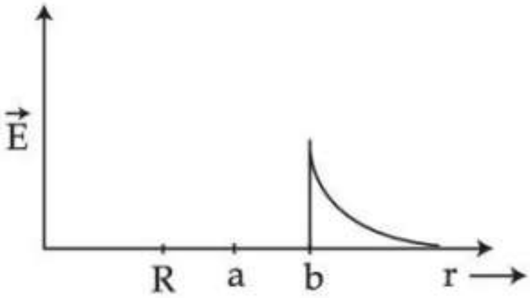


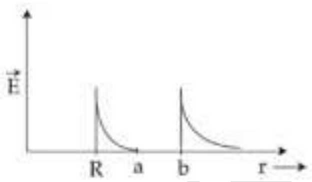
Options

1. 

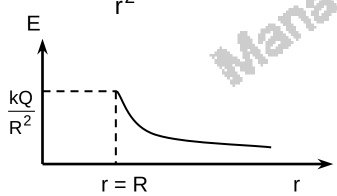
2. 

3. 

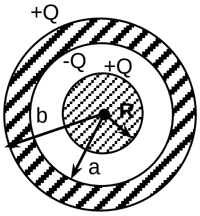
4. 

Ans: 

Sol: If the outer spherical shell is non-conducting
Electric field inside a metal sphere is zero
$r < R \Rightarrow E = 0$
$r > R \Rightarrow E = \frac{kQ}{r^2}$



If the outer spherical shell is conducting

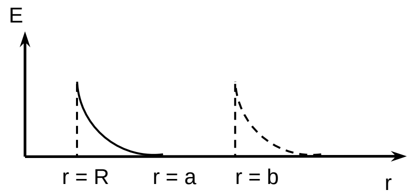


r < R, E = 0

$R \le r < a \qquad E = \frac{kQ}{r^2}$

$a \le r < b \qquad E = 0$

$r \ge b \qquad E = \frac{kQ}{r^2}$



Q.8 An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8 A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :

Options
1. 0.2
2. 0.8
3. 0.4
4. 0.125

Ans: 0.2

Sol: $U = \frac{1}{2}Li^2 = 64 \Rightarrow L = 2$

$i^2R = 640$

$R = \frac{640}{(8)^2} = 10$

$\tau = \frac{L}{R} = \frac{1}{5} = 0.2$

Q.9 The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H, V_O and V_C respectively then :

Options
1. $V_H > V_O > V_C$
2. $V_H = V_O > V_C$
3. $V_H = V_O = V_C$
4. $V_C > V_O > V_H$

Ans: $V_H > V_O > V_C$

Sol: $V_{RMS} = \sqrt{\frac{3RT}{M_W}}$

When temperature is same $V_{RMS} \propto \frac{1}{\sqrt{M_W}}$

$\Rightarrow V_H > V_O > V_C$

Q.10 In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function $\phi = 2.5$ eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. $(h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms$^{-1})$

Options
1. 0.6 V
2. 1.1 V
3. 1.9 V
4. 1.3 V

Ans: 1.3 V

Sol: $KE_{max} = eV_s = \frac{hc}{\lambda} - \phi$

$\Rightarrow eV_s = \frac{1240}{280} - 2.5 = 1.93 eV$

$\rightarrow V_{s_1} = 1.93 V ----- (i)$

$\rightarrow eV_{s_2} = \frac{1240}{400} - 2.5 = 0.6 eV$

$\Rightarrow V_{s_2} = 0.6$ V -----------(ii)

$\Delta V = V_{s_1} - V_{s_2} = 1.93 - 0.6 = 1.33 V$

Q.11 A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from $n=3$ to $n=1$. The frequency in Hz of radiation emitted in transition from $n=2$ to $n=1$ will be :

Options
1. 2.46×10^{15}
2. 6.57×10^{15}
3. 4.38×10^{15}
4. 0.44×10^{15}

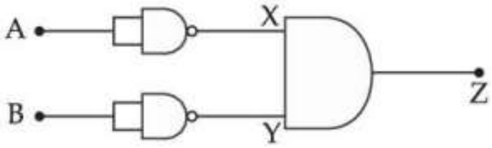
Ans: 2.46×10^{15}

Sol: $nf_1 = k\left(\frac{1}{1} - \frac{1}{3^2}\right)$

$nf_2 = k\left(1 - \frac{1}{2^2}\right)$

$\frac{f_1}{f_2} = \frac{8/9}{3/4} \Rightarrow f_2 = 2.46 \times 10^{15}$

Q.12 Identify the logic operation carried out by the given circuit :



Options
1. NOR
2. AND
3. OR
4. NAND

Ans: NOR

Sol:

A	B	X	Y	Z
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

Q.13 Inside a uniform spherical shell :

(a) the gravitational field is zero.

(b) the gravitational potential is zero.

(c) the gravitational field is same everywhere.

(d) the gravitation potential is same everywhere.

(e) all of the above

Choose the **most appropriate** answer from the options given below :

Options

1. (e) only
2. (a), (b) and (c) only
3. (b), (c) and (d) only
4. (a), (c) and (d) only

Ans: (a), (c) and (d) only

Sol: Inside a spherical shell, gravitational field is zero and the gravitational potential remains same everywhere.

Q.14 The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :

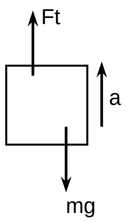
[Use $g = 10\ m/s^2$]

Options

1. 6.0×10^2 kg s^{-1}
2. 60 kg s^{-1}
3. 500 kg s^{-1}
4. 10 kg s^{-1}

Ans: 60 kg s^{-1}

Sol:



$$F_{thrust} = \left(\frac{dm}{dt}V_{rel}\right)$$

$$\left(\frac{dm}{dt}V_{rel} - mg\right) = ma$$

$$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$$

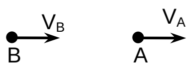
$$\frac{dm}{dt} = (60 kg/s)$$

Q.15 Car B overtakes another car A at a relative speed of $40\ ms^{-1}$. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A ?

Options
1. $40\ ms^{-1}$
2. $0.1\ ms^{-1}$
3. $0.2\ ms^{-1}$
4. $4\ ms^{-1}$

Ans: $0.1\ ms^{-1}$

Sol:



Rear view mirror is used here

$$\therefore V_{I/m} = -m^2 V_{O/m}$$

Given,

$$V_{O/m} = 40 m/s$$

$$m = \frac{f}{f-u} = \frac{10}{10+190} = \frac{10}{200}$$

$$\therefore V_{I/m} = -\frac{1}{400} \times 40 = -0.1 m/s$$

Image of car B appear to move with speed 0.1 m/s

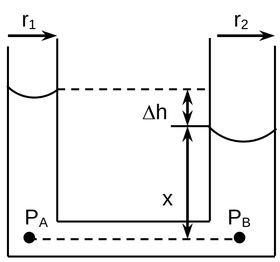
Q.16 Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube.

[Take surface tension of water $T = 7.3 \times 10^{-2}\ Nm^{-1}$, angle of contact = 0, $g = 10\ ms^{-2}$ and density of water $= 1.0 \times 10^3\ kg\ m^{-3}$]

Options
1. 2.19 mm
2. 5.34 mm
3. 3.62 mm
4. 4.97 mm

Ans: 2.19 mm

Sol:



We have $P_A = P_B$ [points A and B at same horizontal level]

$$\therefore\ P_{atm} - \frac{2T}{r_1} + \rho g(x + \Delta h) = P_{atm} - \frac{2T}{r_2} + \rho g x$$

$$\therefore \rho g \Delta h = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= 2 \times 7.3 \times 10^{-2}\left[\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}}\right]$$

$$\therefore \Delta h = \frac{2 \times 7.3 \times 10^{-2} \times 10^3}{10^3 \times 10}\left[\frac{1}{2.5} - \frac{1}{4}\right] = 2.19 \times 10^{-3} \text{ m} = 2.19 \text{ mm}$$

Q.17 If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :

Options
1. $[M^0 L^1 T^0]$
2. $[M^0 L^0 T^0]$
3. $[M^1 L^1 T^{-2}]$
4. $[M^{-1} L^{-1} T^2]$

Ans: $[M^0 L^0 T^0]$

Sol: $E = ML^2T^{-2}$
$L = ML^2T^{-1}$
$m = M$
$G = M^{-1}L^{+3}T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

Q.18 **Statement I :**
By doping silicon semiconductor with pentavalent material, the electrons density increases.
Statement II :
The n-type semiconductor has net negative charge.
In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. Both **Statement I** and **Statement II** are true.
2. Both **Statement I** and **Statement II** are false.
3. **Statement I** is false but **Statement II** is true.
4. **Statement I** is true but **Statement II** is false.

Ans: Statement I is true but Staement II is false

Sol: Pentavalent impurities have excess number of free e^-. but the overall semiconductor will be changeless or neutral

Q.19 What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3 Ω ?

(Given resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively)

Options
1. 82 m
2. 110 m
3. 97 m
4. 90 m

Ans: 97 m

Sol: $\frac{R_1R_2}{R_1+R_2}=3$

$$\frac{\frac{(12\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}}\times\frac{(51\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}}}{\frac{63\times10^{-6}\times10^{-2}\times\ell\times4}{\pi(2)^2\times10^{-6}}}$$

$\Rightarrow \ell = 97$ m

Q.20 The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is : (Take r < a).

Options
1. $\frac{3}{2}\frac{a^2}{r^2}$
2. $\frac{3}{2}\frac{r^2}{a^2}$
3. $\frac{2}{3}\frac{a^2}{r^2}$
4. $\frac{2}{3}\frac{r^2}{a^2}$

Ans: $\frac{3}{2}\frac{r^2}{a^2}$

Sol: $B_{axis}=\frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}$

$B_{centre}=\frac{\mu_0 i}{2R}$

$\therefore B_{centre}=\frac{\mu_0 i}{2a}$

$\therefore B_{axis}=\frac{\mu_0 ia^2}{2(a^2+r^2)^{3/2}}$

$$\therefore \text{ fractional change in magnetic field} = \frac{\frac{\mu_0 i}{2a} - \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}}{\frac{\mu_0 i}{2a}} = 1 - \frac{1}{\left[1+\left(\frac{r^2}{a^2}\right)\right]^{3/2}}$$

$$\approx 1 - \left[1 - \frac{3}{2}\frac{r^2}{a^2}\right] = \frac{3}{2}\frac{r^2}{a^2}$$

Section B

Q.1 Two travelling waves produces a standing wave represented by equation. $y = 1.0 \text{ mm} \cos(1.57 \text{ cm}^{-1})\ x\ \sin(78.5 \text{ s}^{-1})t$. The node closest to the origin in the region $x > 0$ will be at $x =$ _________ cm.

Given --
Answer :

Ans: 1.00

Sol: For node

$\cos(1.57 \text{ cm}^{-1})\ x = 0$

$(1.57 \text{ cm}^{-1})\ x = \frac{\pi}{2}$

$x = \frac{\pi}{2(1.57)} \text{cm} = 1\text{cm}$

Q.2 An amplitude modulated wave is represented by $C_m(t) = 10(1 + 0.2 \cos 12560t) \sin(111 \times 10^4 t)$ volts. The modulating frequency in kHz will be _________.

Given --
Answer :

Ans: 2.00

Sol: $W_m = 12560 = 2\pi f_m$

$f_m = \frac{12560}{2\pi} = 2000 \text{ Hz}$

Q.3 A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be _________ Hz.

Given --
Answer :

Ans: 2025

Sol:

$V_s = 20$ m/s $\quad V_O = 20$ m/s

$f' = f\left(\frac{C - V_0}{C + V_s}\right)$

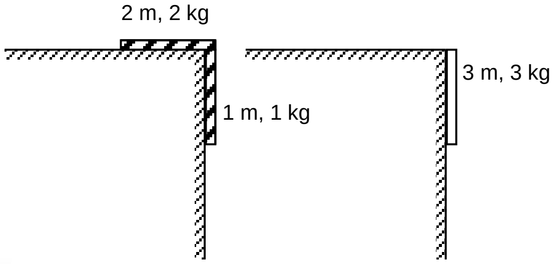
$1800 = f\left(\frac{340 - 20}{340 + 20}\right)$

f = 2025 Hz

Q.4 A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ________.
(Take $g = 10\ m/s^2$)

Given **3**
Answer :

Ans: 40.00



Sol: From energy conservation

$K_i + U_i = k_f + U_f$

$$0 + \left(-1 \times 10 \times \frac{1}{2}\right) = k_f + \left(-3 \times 10 \times \frac{3}{2}\right)$$

$-5 = k_f - 45$

$K_f = 40\ J$

Q.5 The electric field in a plane electromagnetic wave is given by

$$\vec{E} = 200 \cos\left[\left(\frac{0.5 \times 10^3}{m}\right)x - \left(1.5 \times 10^{11}\ \frac{rad}{s} \times t\right)\right] \frac{V}{m}\ \hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\ \frac{N}{m^2}$. Find the value of x.

Given **2**
Answer :

Ans: 354.0

Sol: $E_0 = 200$

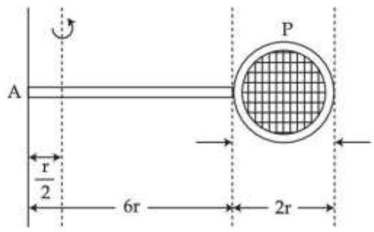
$$I = \frac{1}{2}\varepsilon_0 E_0^2 C$$

Radiation pressure

$$P = \frac{2I}{C} = \left(\frac{2}{C}\right)\left(\frac{1}{2}\varepsilon_0 E_0^2 C\right) = \varepsilon_0 E_0^2 = 8.85 \times 10^{-12} \times 200^2$$

$$= 8.85 \times 10^{-8} \times 4 = \frac{354}{10^9}$$

Q.6 Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be ________ Mr^2.

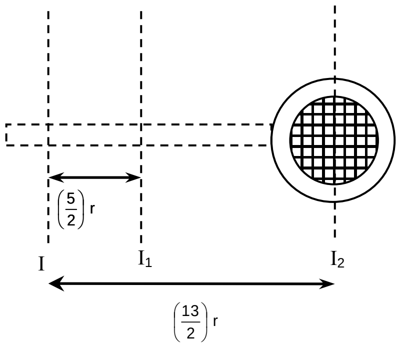
Given **12**
Answer :

Ans: 52.00

Sol: $I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$

$= \left[\frac{M(36r^2)}{12} + \frac{M(25r^2)}{4}\right] + \left[\frac{Mr^2}{2} + \frac{169Mr^2}{4}\right]$

$= 52\ Mr^2$



Q.7 White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and voilet light is _________ nm.

Given --
Answer :

Ans: 300.00

Sol: Position of bright fringe $y = n\frac{D\lambda}{d}$

y_1 of red = $\frac{D\lambda}{d} = 3.5$ mm

$\lambda_r = 3.5 \times 10^{-3}\ \frac{d}{D}$

Similarly $\lambda_v = 2 \times 10^{-3}\frac{d}{D}$

$\lambda_r - \lambda_v = (1.5 \times 10^{-3})\left(\frac{0.3 \times 10^{-3}}{1.5}\right)$

$= 3 \times 10^{-7} = 300$ nm

Q.8 A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is _________ cm.

Given **5**
Answer :

Ans: 2.00

Sol: Excess pressure inside the smaller soap bubble

$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2}$ ------(i)

The excess pressure inside the equivalent soap bubble

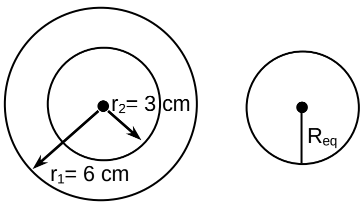
$\Delta P = \frac{4S}{R_{eq}}$ -----(ii)

From (i) and (ii)

$\frac{4S}{R_{eq}} = \frac{4S}{r_1} + \frac{4S}{r_2}$

$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{6} + \frac{1}{3}$

$R_{eq} = 2$ cm

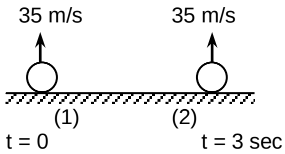


Q.9 Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3 s with the same initial velocity of 35 m/s, then these balls collide at a height of _________ m.
(take $g = 10\ m/s^2$)

Given **30**
Answer :

Ans: 50.00

Sol: When both balls will collide



$y_1 = y_2$

$$35t - \frac{1}{2}\times 10\times t^2 = 35(t-3) - \frac{1}{2}\times 10\times (t-3)^2$$

$$35t - \frac{1}{2}\times 10\times t^2 = 35 - \frac{1}{2}\times 10\times t^2$$

$$-\frac{1}{2}\times 10\times 3^2 + \frac{1}{2}\times 10\times 6t$$

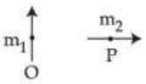
0 = 150 – 30 t
t = 5 sec
∴ Height at which both balls will collide

$$h = 35t - \frac{1}{2}\times 10\times t^2$$

$$= 35\times 5 - \frac{1}{2}\times 10\times 5^2$$

h = 50 m

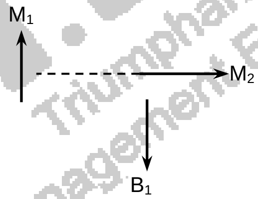
Q.10 Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is _________ $\times 10^{-7}$ Nm.



Given --
Answer :

Ans: 1.00

Sol: $\vec{\tau} = \vec{M}_2 \times \vec{B}_1$



$\tau = M_2 B_1 \sin 90°$

$$= 1\times \frac{\mu_0}{4\pi}\frac{M_1}{(1)^3}1$$

$= 10^{-7}$ Nm

PART – B – CHEMISTRY

Section A

Q.1 Given below are two statements.

Statement I : The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II : The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. Both **Statement I** and **Statement II** are true
2. Both **Statement I** and **Statement II** are false
3. **Statement I** is true but **Statement II** is false
4. **Statement I** is false but **Statement II** is true

Ans: 3

Sol: Statement I is true but statement II is false

Q.2 Given below are two statements :

Statement I : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II : According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

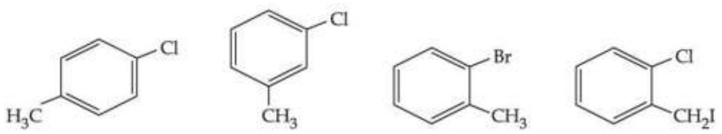
Options
1. Both **Statement I** and **Statement II** are false
2. **Statement I** is false but **Statement II** is true
3. **Statement I** is true but **Statement II** is false
4. Both **Statement I** and **Statement II** are true

Ans: Statement I is false but Statement II is false

Sol: Velocity of an electron in Bohr's atom is given by $V \propto \frac{z}{n}$

Where z → atomic number, which corresponds to number of protons (+ve charges)
Hence, as 'z' increases, velocity increases
As 'n' increases, velocity decreases

Q.3 Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) $AgNO_3$?

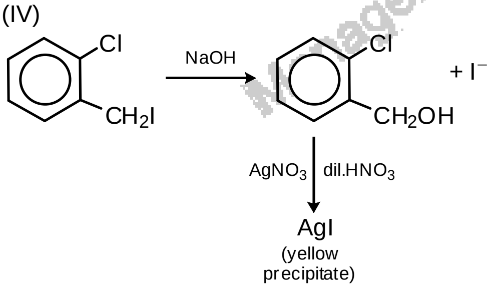


I II III IV

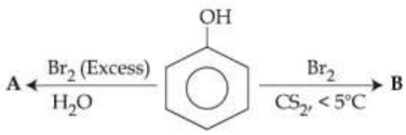
Options
1. IV
2. II
3. I
4. III

Ans: (IV)

Sol:



Q.4 The correct options for the products **A** and **B** of the following reactions are :

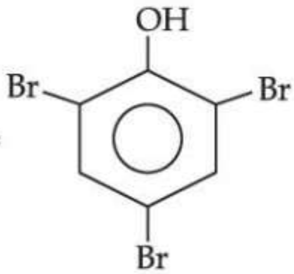


Options 1.

A = 

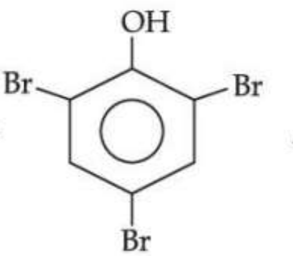
, **B** = 

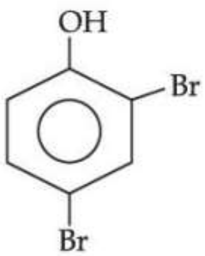
2.

A = 

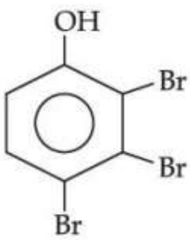
, **B** = 

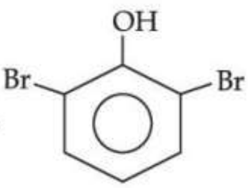
3.

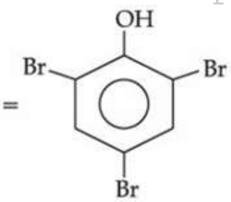
A = 

, **B** = 

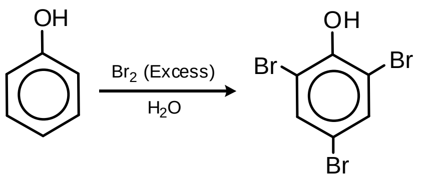
4.

A = 

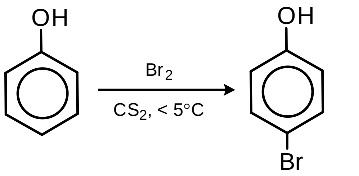
, **B** = 

Ans: **A** = 

, **B** = 

Sol: 

(A)
2,4,6-tribromophenol



(B)
p-Bromophenol

Q.5 The **incorrect** statement is :

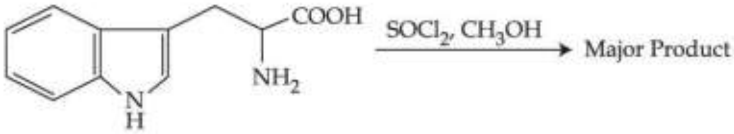
Options

1. F_2 is more reactive than ClF.
2. Cl_2 is more reactive than ClF.
3. On hydrolysis ClF forms HOCl and HF.
4. F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution.

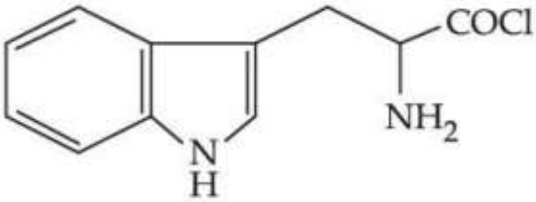
Ans: F_2 is more reactive than ClF

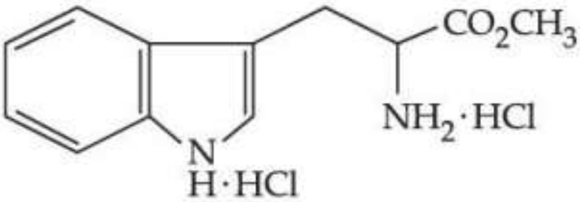
Sol: Interhalogen compounds are more reactive than respective halogens (except fluorine)

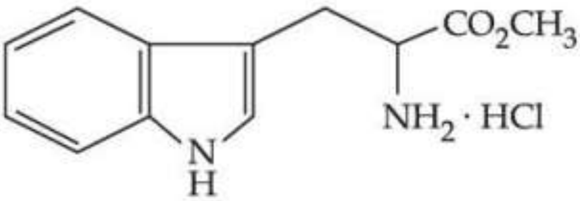
Q.6 The major product formed in the following reaction is :

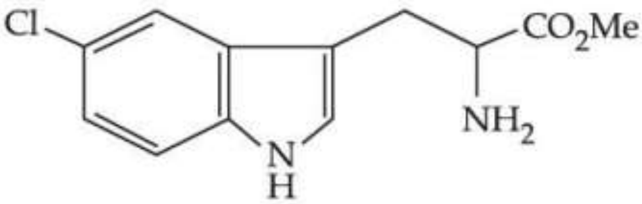


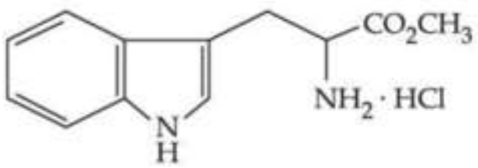
Options

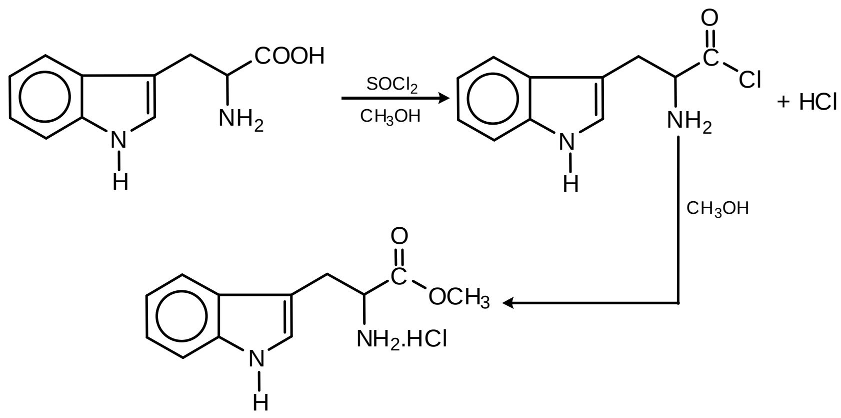
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.7 What are the products formed in sequence when excess of CO_2 is passed in slaked lime ?

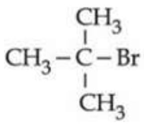
Options
1. $CaO, CaCO_3$
2. $CaCO_3, Ca(HCO_3)_2$
3. $Ca(HCO_3)_2, CaCO_3$
4. $CaO, Ca(HCO_3)_2$

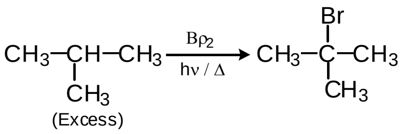
Ans: $CaCO_3$, $Ca(HCO_3)_2$

Sol: $Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 + H_2O \xrightarrow{\text{excess } CO_2} Ca(HCO_3)_2$

Q.8 Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product ?

Options
1. $CH_3-C(CH_3)(Br)-CH_3$ (i.e. $CH_3-\underset{CH_3}{\overset{CH_3}{C}}-Br$)
2. $CH_3-\underset{CH_3}{CH}-CH_2Br$
3. $CH_3-\underset{CH_3}{\overset{Br}{C}}-CH_2-Br$
4. $CH_3-\underset{CH_2Br}{CH}-CH_2Br$

Ans: 

Sol: 

Q.9 Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere ?

Options
1. $CuCl_2$
2. $AgCl$
3. $ZnCl_2$
4. Cu_2Cl_2

Ans: $CuCl_2$

Sol: $CuCl_2$ when dissolved in water forms bluish-green coloured solution.

Q.10 Given below are two statements :

Statement I : In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II : For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. **Statement I** is false but **Statement II** is true
2. **Statement I** is true but **Statement II** is false
3. Both **Statement I** and **Statement II** are false
4. Both **Statement I** and **Statement II** are true

Ans: Statement I is true but statement II is false

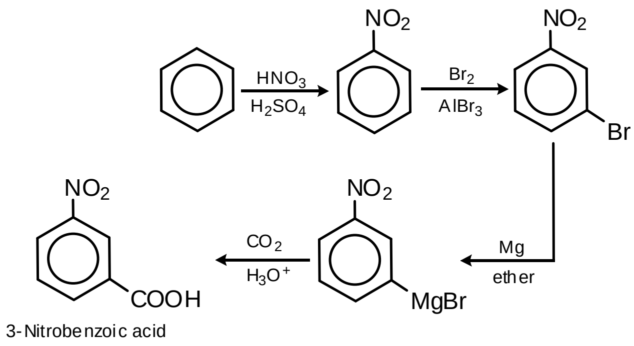
Sol: For weak acid against strong base, indicator phenolphthalein is used and for a strong acid against weak base, indicator methyl orange is used.

Q.11 The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is :

Options
1. $Br_2/AlBr_3$, $NaCN$, H_3O^+, HNO_3/H_2SO_4
2. $Br_2/AlBr_3$, HNO_3/H_2SO_4, Mg/ether, CO_2, H_3O^+
3. HNO_3/H_2SO_4, $Br_2/AlBr_3$, Mg/ether, CO_2, H_3O^+
4. $Br_2/AlBr_3$, HNO_3/H_2SO_4, $NaCN$, H_3O^+

Ans: HNO_3 / H_2SO_4, Br_2 / $AlBr_3$, Mg / ether, CO_2, H_3O^+

Sol:



Q.12 Which one of the following complexes is violet in colour ?

Options

1. $[Fe(SCN)_6]^{4-}$
2. $[Fe(CN)_5NOS]^{4-}$
3. $[Fe(CN)_6]^{4-}$
4. $Fe_4[Fe(CN)_6]_3 \cdot H_2O$

Ans: $[Fe(CN)_5NOS]^{4-}$

Sol: $S^{2-} + \underset{\text{nitro prusside}}{[Fe(CN)_5NO]^{2-}} \longrightarrow \underset{\text{violet}}{[Fe(CN)_5NOS]^{4-}}$

Q.13 The polymer formed on heating Novolac with formaldehyde is :

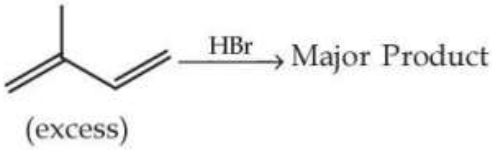
Options

1. Bakelite
2. Polyester
3. Melamine
4. Nylon 6,6

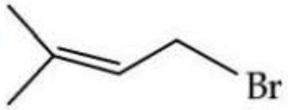
Ans: Bakelite

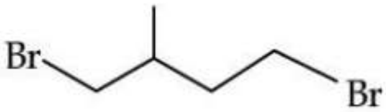
Sol: Novolac on heating with formaldehyde undergoes cross linking to form an infusible solid called bakelite.

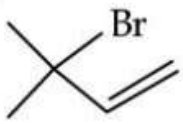
Q.14 The major product formed in the following reaction is :

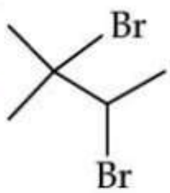


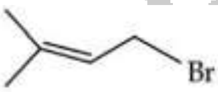
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: $CH_2{=}C(CH_3){-}CH{=}CH_2 \xrightarrow[\text{1,4 addition takes place}]{HBr} CH_3{-}C(CH_3){=}CH{-}CH_2Br$

Q.15 The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is :

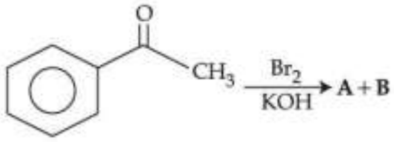
Options

1. $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$
2. $[3Ca_3(PO_4)_2 \cdot CaF_2]$
3. $[3Ca(OH)_2 \cdot CaF_2]$
4. $[Ca_3(PO_4)_2 \cdot CaF_2]$

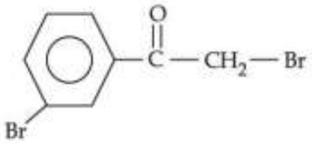
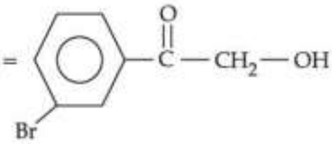
Ans: $[3Ca_3(PO_4)_2.Ca(OH)_2]$

Sol: Hydroxyaptatite is $[3Ca_3(PO_4)_2.Ca(OH)_2]$

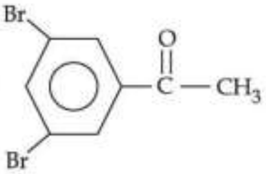
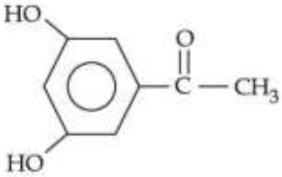
Q.16 The major products formed in the following reaction sequence **A** and **B** are :



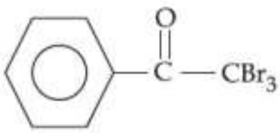
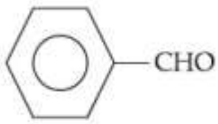
Options 1.

A =  , **B** = 

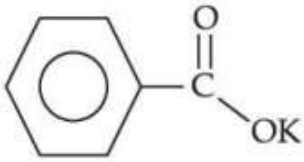
2.

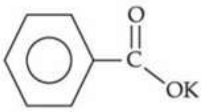
A =  , **B** = 

3.

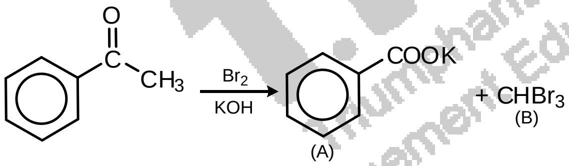
A =  , **B** = 

4.

A =  , **B** = $CHBr_3$

Ans: **A** =  , **B** = $CHBr_3$

Sol:



Q.17 Given below are two statements :

Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. Both **Statement I** and **Statement II** are false
2. **Statement I** is true but **Statement II** is false
3. **Statement I** is false but **Statement II** is true
4. Both **Statement I** and **Statement II** are true

Ans: Both Statement I and statement II are false

Sol: Limiting molar conductivity of KCl is less than that of acetic acid. Molar conductivity increases with increase in dilution (i.e., decrease in concentration)

Q.18 Which one of the following methods is most suitable for preparing deionized water ?

Options
1. Permutit method
2. Synthetic resin method
3. Calgon's method
4. Clark's method

Ans: Synthetic resin method

Sol: Organic ion exchange method (synthetic resin method) is used for preparing deionised water.

Q.19 Given below are two statements :

Statement I : Frenkel defects are vacancy as well as interstitial defects.

Statement II : Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the **most appropriate** answer for the statements from the options given below :

Options
1. Both **Statement I** and **Statement II** are true
2. **Statement I** is true but **Statement II** is false
3. **Statement I** is false but **Statement II** is true
4. Both **Statement I** and **Statement II** are false

Ans: Statement I is true but statement II is false

Sol: Statement I is true but statement II is false

Q.20 Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface ?

Options
1. $\Delta H > 0, \Delta S < 0$
2. $\Delta H > 0, \Delta S > 0$
3. $\Delta H < 0, \Delta S < 0$
4. $\Delta H < 0, \Delta S > 0$

Ans: $\Delta H < 0, \Delta S < 0$

Sol: For adsorption of gas at a given temperature, $\Delta H = -ve$, since it is an exothermic process and $\Delta S = -ve$, since randomness decreases during adsorption.

Section B

Q.1 These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is ________.
(Integer answer)

Given Answer : **2**

Ans: 3

Sol: $M_{(s)} \xrightarrow{\Delta_{sub}H} M_{(g)} \xrightarrow{IE} M^{+}_{(g)} \xrightarrow{\Delta_{hyd}H} M^{+}_{(aq)}$

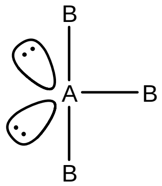
Electrode potential depends on sublimation enthalpy, ionization enthalpy and hydration enthalpy.

Q.2 AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _________. (Integer answer)

Given Answer : **2**

Ans: 2

Sol: For an interhalogen compound AB_3, the central atom A undergoes sp^3d hybridization and there are 3 bond pairs and 2 lone pairs around it.



Q.3 The number of $4f$ electrons in the ground state electronic configuration of Gd^{2+} is _________.
[Atomic number of Gd = 64]

Given Answer : **0**

Ans: 7

Sol: Gd (Z = 64)
Electronic configuration − [Xe] $4f^7\ 5d^1\ 6s^2$
$\therefore Gd^{2+} \rightarrow$ [Xe] $4f^7\ 5d^1$

Q.4 The following data was obtained for chemical reaction given below at 975 K.

$$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$$

	[NO] mol L^{-1}	[H_2] mol L^{-1}	Rate mol $L^{-1}s^{-1}$
(A)	8×10^{-5}	8×10^{-5}	7×10^{-9}
(B)	24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C)	24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is _________. [Integer answer]

Given Answer : **1**

Ans: 1

Sol: Rate = $k[NO]^x [H_2]^y$
According to (A) → $7 \times 10^{-9} = [8 \times 10^{-5}]^x [8 \times 10^{-5}]^y$
According to (B) → $2.1 \times 10^{-8} = k[24 \times 10^{-5}]^x [8 \times 10^{-5}]^y$
According to (C) → $8.4 \times 10^{-8} = k[24 \times 10^{-5}]^x [32 \times 10^{-5}]^y$

Equation (2) ÷ (1) ⇒ $\dfrac{2.1\times10^{-8}}{7\times10^{-9}} = \dfrac{(24\times10^{-5})^x}{(8\times10^{-5})^x}$

$3 = 3^x$ $\quad$ x = 1
Order with respect to NO is 1

Q.5 The ratio of number of water molecules in Mohr's salt and potash alum is _________ $\times 10^{-1}$. (Integer answer)

Given Answer : **5**

Ans: 5

Sol: Mohr's salt is $FeSO_4.(NH_4)_2SO_4.6H_2O$
Potash alum is $K_2SO_4.Al_2(SO_4)_3.12H_2O$

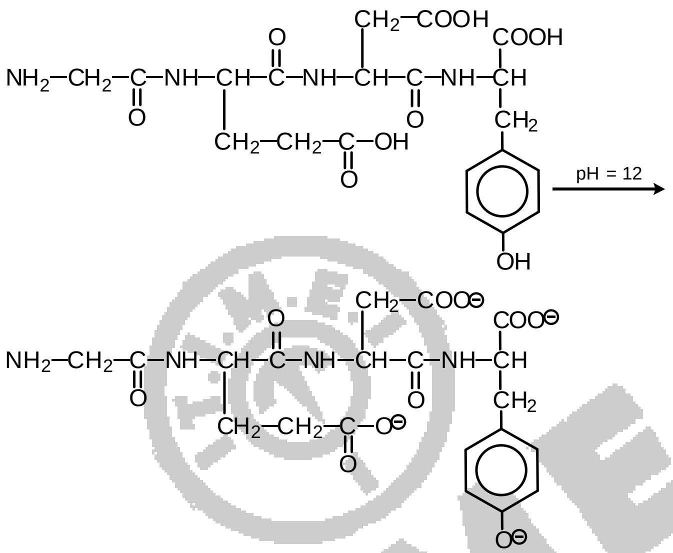
$\therefore$ Ratio of H_2O molecules $= \frac{6}{12} = \frac{1}{12} = 5 \times 10^{-1}$

Q.6 The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be ________. (Integer answer)

Given --
Answer :

Ans: 4

Sol:



Q.7 The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H^{\ominus}$ for KCl = −436.7 kJ mol^{-1}; $\Delta_{sub} H^{\ominus}$ for K = 89.2 kJ mol^{-1};

$\Delta_{ionization} H^{\ominus}$ for K = 419.0 kJ mol^{-1}; $\Delta_{electron\ gain} H^{\ominus}$ for $Cl_{(g)}$ = −348.6 kJ mol^{-1};

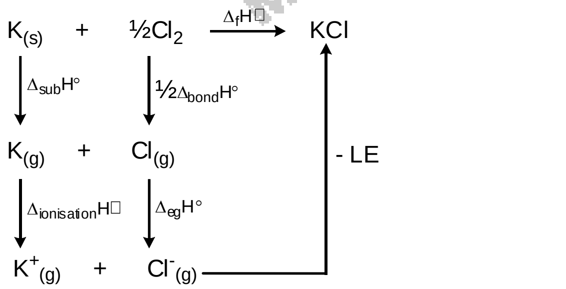
$\Delta_{bond} H^{\ominus}$ for Cl_2 = 243.0 kJ mol^{-1}

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is ________. (Nearest integer)

Given --
Answer :

Ans: 718

Sol:



According to Hess's law

$$\Delta_f H° = \Delta_{sub}H° + \Delta_{ionisation}H° + \frac{1}{2}\Delta_{bond}H° + \Delta_{eg}H° - LE$$

$$\therefore LE = 89.2 + 419.0 + \frac{1}{2} \times 243 + (-348.6) + 436.7$$

$= 717.8\ kJ\ mol^{-1}$

Q.8 Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M C_2H_5OH is ________. (Integer answer)

(i) 0.10 M $Ba_3(PO_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4

Given --
Answer :

Ans: 4

Sol: As $\Delta T_f \uparrow T_f \downarrow$

$\Delta T_f \propto i.m$

For option (i) i × m = 5 × 0.1 = 0.5

For option (ii) i × m = 3 × 0.1 = 0.3

For option (iii) i × m = 2 × 0.1 = 0.2

For option (iv) i × m = 4 × 0.1 = 0.4

As C_2H_5OH is non-dissociative molecule, and rest of all the options are electrolytes, all the options undergo dissociation and hence number of particles increases and therefore freezing point decreases.

Q.9 An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1}. The molarity of the solution in mol L^{-1} is ________. (Nearest integer)
[Molar mass of KCl = 74.5]

Given --
Answer :

Ans: 3

Sol: Molarity = 3.3 mol kg^{-1}

i.e., 3.3 moles of solute → 1 kg solvent

$\therefore\ m_{solvent} = 1000\ g \qquad m_{solute} = 3.3 \times 74.5 = 245.85\ g$

$\therefore m_{soln} = 1245.85\ g \quad d_{soln} = 1.2\ g\ mL^{-1}$

$$\therefore V_{soln} = \frac{M_{soln}}{d_{soln}} = \frac{1245.85}{1.2} = 1038.2\ mL$$

$$\text{Molarity} = \frac{n_{solute}}{V_{soln}\ \text{in L}} = \frac{3.3}{1.0382} = 3.18M$$

Q.10 The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is $x \times 10^{-6}$ M. The value of x is ________. (Nearest integer)
[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

Given --
Answer :

Ans: 3

Sol: $$K_b = \frac{[NH_4^+][OH^-]}{[NH_3]}$$

$$\therefore [OH^-] = \frac{1.8 \times 10^{-5} \times 2 \times 0.021}{5 \times 0.0504} = 3 \times 10^{-6}$$

PART – C – MATHEMATICS

Section A

Q.1 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to :

Options
1. 2
2. 6
3. -2
4. -6

Ans: –3

Sol: $\vec{a} = \hat{i} + \hat{j} + \hat{k} \qquad \vec{b} = \hat{j} - \hat{k}$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{a}.\vec{c} = 3$

$\Rightarrow x + y + z = 3 \qquad (1)$

$$\vec{a} \times \vec{c} = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x)$$

$\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow z = y \; ; \; x = z + 1$

Sub in (1)

$z = \frac{2}{3}, \; y = \frac{2}{3}, \; x = \frac{5}{3}$

$\Rightarrow \vec{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & +1 & -1 \\ \frac{5}{3} & \frac{2}{3} & \frac{2}{3} \end{vmatrix} = \frac{4}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{5}{3}\hat{k}$$

$$\vec{a}.(\vec{b} \times \vec{c}) = (\hat{i} + \hat{j} + \hat{k})\left(\frac{4}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{5}{3}\hat{k}\right) = -2$$

Q.2 The sum of solutions of the equation $\frac{\cos x}{1 + \sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

Options
1. $-\frac{11\pi}{30}$
2. $-\frac{7\pi}{30}$
3. $\frac{\pi}{10}$
4. $-\frac{\pi}{15}$

Ans: $-\frac{11\pi}{30}$

Sol: $\frac{\cos x}{1+\sin x}=|\tan 2x|$

$\Rightarrow \frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}=|\tan 2x|$

$\Rightarrow \frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}=|\tan 2x|$

$\Rightarrow \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)=|\tan 2x|$

$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\tan^2 2x$

$\Rightarrow 2x=n\pi\pm\left(\frac{\pi}{4}-\frac{x}{2}\right)$

$x=\frac{-3\pi}{10},\frac{-\pi}{6},\frac{\pi}{10}$

$\Rightarrow$ Sum of solutions $=\frac{-3\pi}{10}-\frac{\pi}{6}+\frac{\pi}{10}=\frac{-11\pi}{30}$

Q.3 Let ABC be a triangle with A $(-3, 1)$ and $\angle ACB=\theta, 0<\theta<\frac{\pi}{2}$. If the equation of the median through B is $2x+y-3=0$ and the equation of angle bisector of C is $7x-4y-1=0$, then $\tan\theta$ is equal to :

Options

1. $\frac{1}{2}$
2. $\frac{3}{4}$
3. $\frac{4}{3}$
4. 2

Ans: $\frac{4}{3}$

Sol: Let P be the midpoint of A and C, where C is (p,q)

Then $P=\left(\frac{p-3}{2},\frac{q+1}{2}\right)$

Since P lies in $2x+y-3=0$, $2\left(\frac{p-3}{2}\right)+\frac{q+1}{2}-3=0$

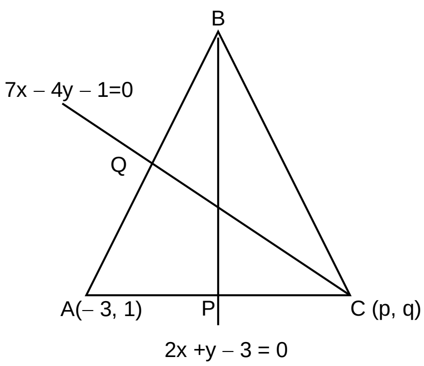
$\Rightarrow 2p+q=11$ ....(1)

Also, (p, q) lies in $7x-4y-1=0$

$\Rightarrow 7p-4q=1$ ...(2)

Solving (1) and (2) we get p = 3, q=5

Now slope of AC = $\frac{5-1}{3-^{-}3}=\frac{2}{3}$



Slope of CQ = $\frac{-7}{-4} = \frac{7}{4}$

$$\angle ACQ = \left|\frac{\frac{2}{3}-\frac{7}{4}}{1+\frac{14}{12}}\right| = \left|\frac{-13}{26}\right| = \frac{1}{2} = \tan\frac{\theta}{2}$$

Now $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2.\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$

Q.4 Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which P (exactly one of A, B occurs) $= \frac{5}{9}$, is :

Options

1. $\frac{1}{3}$
2. $\frac{4}{9}$
3. $\frac{2}{9}$
4. $\frac{5}{12}$

Ans: $\frac{5}{12}$

Sol: A and B are independent $\Rightarrow p(A \cap B) = p(A).p(B)$

p(exactly one of A, B occurs) $\Rightarrow p(A \cap \overline{B}) + p(\overline{A} \cap B)$

$\Rightarrow p(1-2p) + (1-p)2p = \frac{5}{9}$

$\Rightarrow 36P^2 - 27P + 5 = 0$

$\Rightarrow p = \frac{1}{3}, \frac{5}{12}$

$\Rightarrow$ Largest value of p = $\frac{5}{12}$

Q.5

The sum of the series $\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+\ldots+\frac{2^{100}}{x^{2^{100}}+1}$ when $x=2$ is :

Options

1. $1-\frac{2^{100}}{4^{100}-1}$

2. $1-\frac{2^{101}}{4^{101}-1}$

3. $1+\frac{2^{101}}{4^{101}-1}$

4. $1+\frac{2^{100}}{4^{101}-1}$

Ans: $1-\frac{2^{101}}{4^{101}-1}$

Sol: Let $S=\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+------+\frac{2^{100}}{x^{2^{100}}+1}$

$$S-\frac{1}{1-x}=\frac{-1}{1-x}+\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+------+\frac{2^{100}}{x^{2^{100}}+1}$$

$$=\frac{2}{x^2-1}+\frac{2}{x^2+1}+\frac{2^2}{x^4+1}+------+\frac{2^{100}}{x^{2^{100}}+1}$$

$$=\frac{2^2}{x^4-1}+\frac{2^2}{x^4+1}+------+\frac{2^{100}}{x^{2^{100}}+1}$$

$$\Rightarrow S-\frac{1}{1-x}=\frac{-2^{101}}{x^{2^{101}}-1}$$

Putting x = 2

$$S-\frac{1}{1-2}=\frac{-2^{101}}{4^{101}-1}$$

$$S=\frac{-2^{101}}{4^{101}-1}+1=1-\frac{2^{101}}{4^{101}-1}$$

Q.6

Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, $0<x<1$. Then :

Options

1. $(1+x)^2 f'(x)-2(f(x))^2=0$

2. $(1-x)^2 f'(x)-2(f(x))^2=0$

3. $(1+x)^2 f'(x)+2(f(x))^2=0$

4. $(1-x)^2 f'(x)+2(f(x))^2=0$

Ans: $(1-x)^2 f'(x)+2(f(x))^2=0$

Sol: $f(x) = \cos\left(2\tan^{-1}\sin\cot^{-1}\sqrt{\frac{1-x}{x}}\right)$

$= \cos\left(2\tan^{-1}\ \sin\sin^{-1}\sqrt{x}\right)$

$= \cos\left(2\tan^{-1}\sqrt{x}\right)$

$= \cos\ \tan^{-1}\frac{2\sqrt{x}}{1-x}$

$= \cos\cos^{-1}\frac{1-x}{1+x}$

$\Rightarrow f(x) = \frac{1-x}{1+x}$

Multiplying by $(1-x)^2$ in both sides

$(1-x)^2 f'(x) = -2\frac{(1-x)^2}{(1+x)^2}$

$\Rightarrow (1-x)^2 f'(x) + 2(f(x))^2 = 0$

Q.7

The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with :

Options

1. centre at (0, 0) and radius $\sqrt{2}$
2. centre at (0, 1) and radius 2
3. centre at (0, 1) and radius $\sqrt{2}$
4. centre at (0, -1) and radius $\sqrt{2}$

Ans: centre at (0, 1) and radius $\sqrt{2}$

Sol: Putting $z = x + iy$

$\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1} = \frac{x^2 - 1 - i(x-1)y + i(x+1)y + y^2}{(x+1)^2 + y^2}$

$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$

$\Rightarrow \frac{2y}{x^2+y^2-1} = 1$

$\Rightarrow x^2 + y^2 - 2y - 1 = 0$

$\Rightarrow$ a circle with centre at (0, 1) and radius $\sqrt{2}$

Q.8 Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\, dx + \tan x\, dy + y dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x\to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

Options

1. $\frac{\pi}{4} + 1$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{4} - 1$
4. $-\frac{\pi}{4}$

Ans: $\frac{\pi}{4}$

Sol: Rearranging the given differential equation, we get

$$\frac{dy}{dx} + y(\tan x + \cot x) = -\tan x$$

$$IF = e^{\int(\tan x + \cot x)dx} = \tan x$$

Solution is $y \tan x = \int -\tan^2 x + C$

$$\Rightarrow y \tan x = \int (1 - \sec^2 x)dx + C$$

$$\Rightarrow y \tan x = x - \tan x + C$$

$$\lim_{x\to 0^+} xy = 1 \Rightarrow \lim_{x\to 0^+}\left(\frac{x}{\tan x}\right)(x - \tan x + C) = 1 \Rightarrow C = 1$$

$$\therefore y \tan x = x - \tan x + 1$$

At $x = \frac{\pi}{4}, y = \frac{\pi}{4} - 1 + 1 = \frac{\pi}{4}$

Q.9

The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2 + 4r^2}$ is :

Options

1. $\frac{1}{4}\tan^{-1}(4)$
2. $\frac{1}{2}\tan^{-1}(2)$
3. $\frac{1}{2}\tan^{-1}(4)$
4. $\tan^{-1}(4)$

Ans: $\frac{1}{2}\tan^{-1}(4)$

Sol: $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}=\lim_{n\to\infty}\frac{1}{n}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$=\int_0^2\frac{1}{1+4x^2}dx=\frac{1}{4}\int_0^2\frac{1}{\frac{1}{4}+x^2}dx=\frac{1}{4}2\left[\tan^{-1}2x\right]_0^2=\frac{1}{2}\tan^{-1}4$

Q.10 If a line along a chord of the circle $4x^2+4y^2+120x+675=0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2=30x$, then the length of this chord is :

Options
1. $5\sqrt{3}$
2. 5
3. $3\sqrt{5}$
4. 7

Ans: $3\sqrt{5}$

Sol: Given circle is $4x^2+4y^2+120x+675=0$

$\Rightarrow$ Centre of the circle is $(-15,0)$

Equation of the tangent to the parabola $y^2=30x$ is $y=mx+\frac{30}{4m}$

Since it passes through $(-30,0)$

$4m^2=1\Rightarrow m=\pm\frac{1}{2}$

Case 1 : when $m=\frac{1}{2}$

$y=\frac{x}{2}+15$

$\Rightarrow x-2y+30=0$

Distance from $(-15,0)=\frac{15+0+30}{\sqrt{5}}=3\sqrt{5}$

Radius of the circle $=\frac{15}{2}$

$\therefore$ Half length of chord = $\sqrt{\frac{225}{2}-45}=\frac{3\sqrt{5}}{2}$

Length of chord = $3\sqrt{5}$

Case 2: $m=-\frac{1}{2}$

$y=-\frac{x}{2}-15$

$\Rightarrow x+2y+30=0$

Distance from $(-15,0)=\frac{-15+0+30}{\sqrt{5}}=3\sqrt{5}$

Radius of the circle $=\frac{15}{2}$

Length of chord = $3\sqrt{5}$

$\therefore$ In both cases, length of chord = $3\sqrt{5}$

Q.11

Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations.

$(1+\cos^2\theta)\ x + \sin^2\theta\ y + 4\ \sin 3\theta\ z = 0$

$\cos^2\theta\ x + (1+\sin^2\theta)\ y + 4\ \sin 3\theta\ z = 0$

$\cos^2\theta\ x + \sin^2\theta\ y + (1+4\ \sin 3\theta) z = 0$

has a non-trivial solution, then the value of θ is :

Options

1. $\frac{4\pi}{9}$
2. $\frac{\pi}{18}$
3. $\frac{5\pi}{18}$
4. $\frac{7\pi}{18}$

Ans: $\frac{7\pi}{18}$

Sol: System has non trivial solution

$\Rightarrow |A| = 0$

$$\Rightarrow \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & (1+4\sin 3\theta) \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & (1+4\sin 3\theta) \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0$$

Expanding using row 2

$2 + 4\sin 3\theta = 0$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

Q.12 On the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x+2y=0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5-e^2)\cdot A$ is :

Options
1. 14
2. 12
3. 24
4. 6

Ans: 6

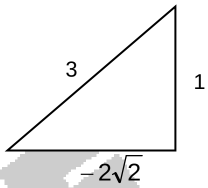
Sol: Given ellipse is $\frac{x^2}{8}+\frac{y^2}{4}=1$

$a=2\sqrt{2}, b=2 \;\Rightarrow e=\frac{1}{\sqrt{2}}$

Let the point P be $(a\cos\theta, b\sin\theta)$

Tangent at this point is given by $\frac{x}{\sqrt{8}}\cos\theta+\frac{y}{2}\sin\theta-1=0$

Slope $=\frac{-1}{\sqrt{2}}\cot\theta=2$



$\Rightarrow \cot\theta=-2\sqrt{2}$

$\cos\theta=\frac{-2\sqrt{2}}{3},\; \sin\theta=\frac{1}{3}$

$\Rightarrow P=\left(2\sqrt{2}\times\frac{-2\sqrt{2}}{3}, 2\times\frac{1}{3}\right)=\left(-\frac{8}{3},\frac{2}{3}\right)$

Area $=\frac{1}{2}\times 2ae\times\frac{2}{3}=\frac{1}{2}\times 2\times 2\sqrt{2}\times\frac{1}{\sqrt{2}}\times\frac{2}{3}=\frac{4}{3}$

Now $(5-e^2)A=\left(5-\frac{1}{2}\right)\frac{4}{3}=6$

Q.13 If the truth value of the Boolean expression $((p\vee q)\wedge(q\rightarrow r)\wedge(\sim r))\rightarrow(p\wedge q)$ is false, then the truth values of the statements p, q, r respectively can be :

Options
1. T F F
2. F F T
3. F T F
4. T F T

Ans: **T F F**

Sol:

p	q	r	$p\vee q$	$q\rightarrow$	**~r**	$(p\vee q)\wedge(q\rightarrow r)\wedge$ **~r**	$p\wedge q$	$((p\vee q)\wedge(q\rightarrow r)\wedge\sim r)\rightarrow p\wedge q$
T	**T**	**T**	**T**	**T**		**F**	**T**	**T**
T	**T**	**F**	**T**	**F**		**F**	**T**	**T**
T	**F**	**T**	**T**	**T**		**F**	**F**	**T**
T	**F**	**F**	**T**	**T**		**T**	**F**	**F**
F	**T**	**T**	**T**	**T**		**F**	**F**	**T**
F	**T**	**F**	**T**	**F**		**F**	**F**	**T**
F	**F**	**T**	**F**	**T**		**F**	**F**	**T**
F	**F**	**F**	**F**	**T**		**F**	**F**	**T**

Q.14

The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{1/2} dx$ is :

Options
1. $\log_e 16$
2. $\log_e 4$
3. $4 \log_e (3 + 2\sqrt{2})$
4. $2 \log_e 16$

Ans: $\log_{e16}$

Sol: $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2\right)^{1/2} dx = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \sqrt{\left(\frac{-4x}{x^2-1}\right)^2} = \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \frac{4|x|}{1-x^2} dx$

$= 8\int_0^{1/\sqrt{2}} \frac{x}{1-x^2} = \left[4\ln\left(1-x^2\right)\right]_0^{1/\sqrt{2}} = \ln 16 = \log_e 16$

Q.15 A plane P contains the line $x+2y+3z+1=0=x-y-z-6$, and is perpendicular to the plane $-2x+y+z+8=0$. Then which of the following points lies on P ?

Options
1. $(1, 0, 1)$
2. $(0, 1, 1)$
3. $(2, -1, 1)$
4. $(-1, 1, 2)$

Ans: $(0,1,1)$

Sol: Equation of the plane containing $x+2y+3z+1=0$ and $x-y-z-6=0$ is

$x+2y+3z+1+\lambda(x-y-z-6)=0$

$\Rightarrow (1+\lambda)x+(2-\lambda)y+(3-\lambda)z+(1-6\lambda)=0$

Since the plane is perpendicular to $-2x+y+z+8=0$

$-2(1+\lambda)+1(2-\lambda)+1(3-\lambda)=0$

$\Rightarrow \lambda = \frac{3}{4}$

$\Rightarrow$ Required plane is $7x+5y+9z=14$

$(0,1,1)$ lie in the plane.

Q.16 If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\ {}^{20}C_r$ is equal to :

Options
1. 380×2^{19}
2. 420×2^{19}
3. 380×2^{18}
4. 420×2^{18}

Ans: 420×2^{18}

Sol: $\sum_{r=0}^{20} r^2\ {}^{20}C_r = \sum_{r=0}^{20}[r(r-1)+r]\ {}^{20}C_r = \sum r(r-1)\frac{20.19}{r(r-1)}\ {}^{18}C_r = \sum r.\frac{20}{r}\ {}^{19}C_r$

$= 20\times 19\times 2^{18} + 20\times 2^{19} = 420\times 2^{18}$

Q.17 If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i=\sqrt{-1}$, and $Q = A^TBA$, then the inverse of the matrix $A\,Q^{2021}\,A^T$ is equal to :

Options
1. $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$
2. $\begin{pmatrix} 1 & 0 \\ 2021\,i & 1 \end{pmatrix}$
3. $\begin{pmatrix} 1 & 0 \\ -2021\,i & 1 \end{pmatrix}$
4. $\begin{pmatrix} 1 & -2021\,i \\ 0 & 1 \end{pmatrix}$

Ans: $\begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

Sol: $AA^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$

$Q = A^TBA$

$Q^2 = (A^TBA)(A^TBA) = A^TB^2A$

$Q^3 = (A^TBA)(A^TB^2A) = A^TB^3A$

$\Rightarrow Q^{2021} = A^TB^{2021}A$

Now, $P = AQ^{2021}A^T$

$\Rightarrow P = A(A^TB^{2021}A)A^T = B^{2021}$

Now, $B^2 = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix}$

$B^3 = \begin{bmatrix} 1 & 0 \\ i & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2i & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3i & 1 \end{bmatrix}$

$B^{2021} = \begin{bmatrix} 1 & 0 \\ 2021i & 1 \end{bmatrix}$

$P^{-1} = \left(B^{2021}\right)^{-1} = \begin{bmatrix} 1 & 0 \\ -2021i & 1 \end{bmatrix}$

Q.18 If the sum of an infinite GP a, ar, ar^2, ar^3, . . . is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, . . . is :

Options

1. $\frac{5}{2}$
2. $\frac{25}{2}$
3. $\frac{9}{2}$
4. $\frac{1}{2}$

Ans: $\frac{1}{2}$

Sol: $\frac{a}{1-r} = 15$ . ....(1)

$\frac{a^2}{1-r^2} = 150$ ....(2)

$\frac{(2)}{(1)} \Rightarrow \frac{a}{1-r} = 10$ ....(3)

Solving (1) and (3)

$r = \frac{1}{5}$, $a = 12$

Now $ar^2 + ar^4 + ar^6 + - - - - - \infty = \frac{ar^2}{1-r^2} = \frac{12 \times \frac{1}{25}}{1 - \frac{1}{25}} = \frac{1}{2}$

Q.19 The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

Options

1. (10.5, 26)
2. (11, 25)
3. (10.5, 25)
4. (11, 26)

Ans: $(10.5, 26)$

Sol: $\frac{\sum x}{20} = 10$

$\Rightarrow \sum x = 200$

Correct $\sum x$ =200-25+35=210

Correct $\bar{x} = \frac{210}{20} = 10.5$

$\frac{\sum x^2}{20} - 100 = 6.25$

$\Rightarrow \sum x^2 = 2125$

Correct $\sum x^2 = 2125 - 625 + 1225 = 2725$

Correct $\sigma^2 = \frac{2725}{20} - (10.5)^2 = 26$

$\Rightarrow \alpha = 10.5$, $\beta = 26$

Q.20 Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can **not** belong to the set :

Options
1. {79, 81, 83, 85}
2. {84, 87, 90, 93}
3. {84, 86, 88, 90}
4. {80, 83, 86, 89}

Ans: $\{79, 81, 83, 85\}$

Sol: $n(H) = 89\%$

$n(L) = 98\%$

$n(H \cap L) = x\%$

Maximum value of x is 89

Minimum value of x is 89 +98 − 100 = 87

So $87 \le x \le 89$

Section B

Q.1 If $y = y(x)$ is an implicit function of x such that $\log_e(x+y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to __________.

Given --
Answer :

Ans: 40

Sol: $\log_e(x+y) = 4xy$; when $x = 0, y = 1$

$\Rightarrow x + y = e^{4xy}$

Differentiating both sides,

$1 + \frac{dy}{dx} = e^{4xy}\left(4y + 4x\frac{dy}{dx}\right)$; when $x = 0$ and $y = 1, \frac{dy}{dx} = 3$

Differentiating again, we get

$$\frac{d^2y}{dx^2} = e^{4xy}\left(4y + 4x\frac{dy}{dx}\right)^2 + e^{4xy}\left(4\frac{dy}{dx} + 4x\frac{d^2y}{dx^2}\right)$$

When $x = 0$, $y = 1$, $\frac{dy}{dx} = 3$

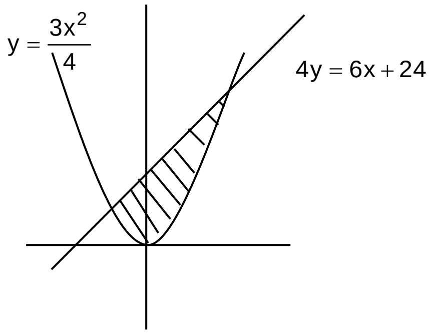
$$\frac{d^2y}{dx^2} = 1(4+0) + 1(8 \times 3 + 0) = 40$$

Q.2 The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is ________.

Given **96**
Answer :

Ans: 27

Sol: Required area is the shaded region



Solving $y = \frac{3x^2}{4}$ and $4y = 6x + 24$

$x = -2,\ x = 4$

Area = $\int_{-2}^{4}\left(\frac{6x+24}{4} - \frac{3x^2}{4}\right)dx$

$= \left(\frac{3}{2}\cdot\frac{x^2}{2} + 6x - \frac{x^3 4}{}\right)_{-2}^{4} = 27$

Q.3 A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi} + 1\right)k$ is equal to ________.

Given --
Answer :

Ans: 36

Sol: Given circumference of circle = k
Let r be its radius

$2\pi r = k$

$r = \frac{k}{2\pi}$

Now circumference of square = 36 – k

Side of square = $9 - \frac{k}{4}$

Area, A = $\pi\frac{k^2}{4\pi^2} + \left(9 - \frac{k}{4}\right)^2 = \frac{k^2}{4\pi} + \left(9 - \frac{k}{4}\right)^2$

$\frac{dA}{dk} = 0 \Rightarrow \frac{2k}{4\pi} + 2\left(9 - \frac{k}{4}\right)\left(-\frac{1}{k}\right) = 0 \Rightarrow k = \frac{36\pi}{\pi + 4}$

$\therefore \left(\frac{4}{\pi} + 1\right)k = 36$

Q.4 The sum of all integral values of k ($k \ne 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is ________.

Given --
Answer :

Ans: 66

Sol: $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$

Rearranging , we get

$2x^2-(6+k)x+3k+4=0$

Since there is no real root,

$(6+k)^2-4.2(3k+4)<0$

$\Rightarrow k^2+12k+36-24k-32<0$

$\Rightarrow k^2+12k+4<0$

$\Rightarrow (k-6)^2-32<0$

$\Rightarrow$ Integral value of k = 1,2,3,……..11

$\text{Sum}=1+2+----+11$

$=\frac{11\times 12}{2}=66$

Q.5 The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to ________.

Given **9**
Answer :

Ans: 16

Sol: Let (x, y) be the point

Then, $x^2+y^2+(x-1)^2+y^2+x^2+(y-1)^2+(x-1)^2+(y-1)^2=18$

$=4x^2+4y^2-4y-4x=14$

$\Rightarrow x^2+y^2-x-y-\frac{7}{2}=0$

$r=\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{7}{2}}=2$

$\Rightarrow d=4\Rightarrow d^2=16$

Q.6 Let $z=\frac{1-i\sqrt{3}}{2}, i=\sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

is __________.

Given --
Answer :

Ans: 13

Sol: $z=\frac{1-i\sqrt{3}}{2}=-\omega$

Now $21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+------+\left(z^{21}+\frac{1}{z^{21}}\right)^3$

$=21+\left(-\omega-\frac{1}{\omega}\right)^3+\left(\omega^2+\frac{1}{\omega^2}\right)^3+-----+\left(-\omega^{21}-\frac{1}{\omega^{21}}\right)^3$ = 21 – 8 = 13

Q.7 If $^1P_1+2\cdot{}^2P_2+3\cdot{}^3P_3+...+15\cdot{}^{15}P_{15}={}^qP_r-s,\ 0\le s\le 1$, then $^{q+s}C_{r-s}$ is equal to ________.

Given --
Answer :

Ans: 136

Sol: $^1P_1+2\ {}^2P_2+3\ {}^3P_3+-------+15\ {}^{15}P_{15}$ = 1!+2 2! + 3 3!+-------+15 15!

$$=\sum_{r=1}^{15} r\,r! = \sum_{r=1}^{15}(r+1-1)r!$$

$$=\sum_{r=1}^{15}(r+1)r!-r! = \sum_{r=1}^{15}(r+1)!-r!$$

$$= 16! - 1!$$

$$= {}^{16}P_{16}-1={}^qP_r-s$$

$\Rightarrow q=16, r=16, s=1$

$\therefore {}^{q+s}C_{r-s}={}^{16+1}C_{16-1}={}^{17}C_{15}={}^{17}C_2=\frac{17\times16}{2}=136$

Q.8 The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

Given **260**
Answer :

Ans: 52

Sol: Even numbers end with 0, 4 or 6
Case(1) ending with 0
5 4 1 = 20
Case(2) ending with 4 or 6
4 4 2 = 32
Total cases = 20 + 32 = 52

Q.9 Let a, b $\in \mathbf{R}$, b $\neq$ 0. Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \frac{\tan 2x-\sin 2x}{b x^3}, & \text{for } x>0. \end{cases}$$

If f is continuous at $x=0$, then $10-ab$ is equal to ________.

Given **4**
Answer :

Ans: 14

Sol: LHL = $\lim_{x\to0} a\sin\left(\frac{\pi}{2}\right)(0-1)=-a$, $f(0)=-a$

RHL= $\lim_{x\to0}\frac{2x-\sin 2x}{bx^3}=\lim_{x\to0}\frac{\frac{(2x)^3}{3}+\frac{(2x)^3}{6}}{bx^3}=\frac{\frac{8}{3}+\frac{8}{6}}{b}=\frac{4}{b}$

$\Rightarrow -a=\frac{4}{b}$

$\Rightarrow -ab=4$

$\therefore 10-ab=10+4=14$

Q.10 Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $x-2y-z=3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to ________.

Given **16**
Answer :

Ans: 26

Sol: Any point in the given line will be of the form $(2k+1, k+3, 2k+4)$

Foot of the perpendicular in the plane is

$$\frac{x-(2k+1)}{1}=\frac{y-(k+3)}{-2}=\frac{z-(2k+4)}{-1}=\frac{k+6}{3}$$

$$\Rightarrow x=\frac{7k+9}{3}, y=\frac{k-3}{3}, z=\frac{5k+6}{3}$$

Projection Line is $\frac{x-3}{7}=\frac{y+1}{1}=\frac{z-2}{5}$

Distance ratio of line joining (0, 0, 6) and projection line is $7k+3, k+1, 5k-4$

Since projection line perpendicular to this , k = 0

$\therefore$ Any arbitrary point in the projection line is $(3,-1,2) \Rightarrow d^2=26$